Measurements of the Frequency Spectrum of Transition Radiation*

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We report a measurement of the frequency spectrum of x-ray transition radiation. X rays were generated by electrons of 5 and 9 GeV in radiators of multiple polypropylene foils, and detected in the range 4 to 30 keV with a calibrated single-crystal Bragg spectrometer. The experimental results closely reproduce the features of the theoretically predicted spectrum. In particular, the pronounced interference pattern of multifoil radiators and the expected hardening of the radiation with increasing foil thickness are clearly observed. The overall intensity of the radiation is somewhat lower than predicted by calculations.

The physical characteristics of x-ray transition radiation have mainly been studied in measurements of the *total* radiation yield. Various radiator-detector configurations[1-8] have led to results which are in fair agreement with the theoretical predictions[9-11] (including the expected saturation of the yield at high particle energies). However, a precise measurement of the frequency *spectrum* of this radiation is a much more significant test of the theory, and, furthermore, leads to details crucial for the design of transition-radiation detectors for relativistic particles. The results of such a measurement are reported in this Letter.

Let us denote the distribution of the transition-radiation intensity with respect to the emission angle θ, and the frequency ω, as $d^2S/d\theta d\omega$. In the idealized case of relativistic particles traversing a *single* interface between two different media, this distribution yields, after integration over angles, a frequency spectrum that is a *monotonic function* of the x-ray energy.[12] In practice, intensity considerations require a radiator of *many interfaces*. The radiator dimensions and the x-ray wavelengths may be comparable in the rest frame of a relativistic particle. Therefore, the coherent addition of the radiation amplitudes from the various interfaces leads to interference phenomena which drastically modify the single-interface radiation pattern[12] $d^2S_0/d\theta d\omega$. For a transparent radiator of N foils of thickness l_1 equally spaced by distances l_2, one obtains[4,9-11,13]

$$\frac{d^2S}{d\theta d\omega} = \frac{d^2S_0}{d\theta d\omega}\, 4\sin^2\!\left(\frac{l_1}{Z_1}\right) \times \frac{\sin^2[N(l_1/Z_1 + l_2/Z_2)]}{\sin^2(l_1/Z_1 + l_2/Z_2)}, \qquad (1)$$

where $Z_{1,2}(\theta,\omega)$ are the "formation zones" of the two media.[14] Using procedures that have been discussed earlier,[4,9-11] we integrate Eq. (1) over angles and take into account possible reabsorption of x rays inside the radiator. The resulting frequency spectrum exhibits striking deviations from that of a single interface: Pronounced maxima and minima occur; and, for sufficiently high particle energy, the spectrum saturates, i.e., it becomes independent of the particle energy. It should be pointed out that such oscillations do occur even in the case of a single foil ($N=1$). If the ratio l_2/l_1 is large (i.e., $l_2/l_1 > 10$), the shape of the spectrum is largely determined by the single-foil interference, although the multiple-foil interference governs the saturation at high energies.[4,13] The exact spectral shape and the positions of maximum intensity are determined by the dimensions of the radiator.[4] Knowledge of these details is of prime importance for practical applications.

In the present experiment, performed at the Cornell University synchrotron, we have measured the transition-radiation spectrum, generated by electrons of 5 and 9 GeV in radiators of evenly spaced polypropylene foils (CH_2), with dimensions (200–1000 foils, thickness 16 to 82 μm, spacing 1.4 mm) that are typical for practical applications. A magnet was used to deflect the electrons away from the x-ray detector. We have used a single-crystal Bragg spectrometer. Thus we avoided an intrinsic difficulty encountered in previous experiments[1,2,15-17] where a proportional or solid-state detector was used to measure the x-ray energy, such that single-photon events became indistinguishable from events due to several simultaneous photons. One such experiment has, however, been quite successful[17] because only very short radiators were used. Both single- and multiple-foil interference phe-

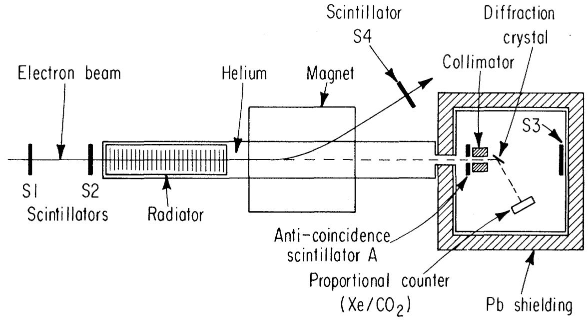

FIG. 1. Diagram of the experimental setup.

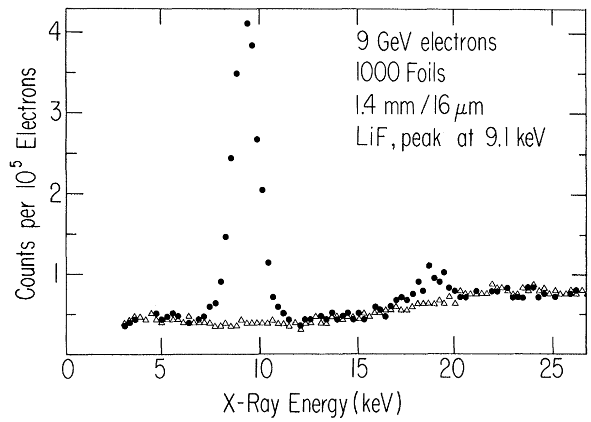

FIG. 2. Measured pulse-height distributions in the proportional counter. The solid points show transition radiation from 1000 CH_2 foils, while the open symbols are obtained with a background radiator. Note the good agreement between the distributions outside the x-ray peak.

nomena were observed in this case.

Figure 1 shows our experimental arrangement. The x-rays reached the spectrometer through a 0.5-cm wide collimator, 6 m from the radiator. The spectrometer consisted of a LiF or an organic PET (polyethylene terephthalate) crystal, covering the energy range 4 to 30 keV, and a xenon proportional counter, the signal of which was pulse-height analyzed. The whole apparatus was kept in a helium-filled enclosure to reduce x-ray absorption. Frequent tests were performed throughout the runs to check the alignment between spectrometer and beam. In order to normalize the measured x-ray intensity to the electron flux, we assumed that transition-radiation photons follow closely (i.e., $\theta \simeq mc^2/E$) the trajectory of their parent electrons in the absence of a magnetic field. This assumption was verified by steering the beam across the spectrometer slit. With the magnet turned off, the flux of parent electrons was determined with scintillators by measuring the rate $S1 \cdot S2 \cdot S3 \cdot \bar{A}$ (see Fig. 1), where A is a scintillator with an opening through the center matching exactly the dimensions of the spectrometer slit.

Figure 2 shows two superimposed pulse-height distributions, measured with the LiF crystal set for 9-keV x rays. One distribution was generated with a radiator in place, while for the other the radiator was replaced by a solid block of polyethylene of the same total mass. This background test has been performed for every run. The distinct peak at 9 keV due to transition radiation is excellently resolved from background, and at 18 keV even a small second-order peak can be noticed. The width of the x-ray peaks is determined by the resolution of the proportional counter. Because of the high resolution of the crystal itself, more than 10^3 electrons are needed for each 9-keV x-ray count. In view of this low differential yield, special efforts were required to shield the apparatus from background.

The absolute reflectivity of the crystals has been calibrated with laboratory x-ray machines. The effective resolution varied between ~1 eV at 4 keV and ~40 eV at 29 keV. Details of these calibration measurements will be described in a separate publication.

In Fig. 3 we show the results of our measurements for three radiators with different foil thicknesses, normalized as photons per interface and per electron. The number of foils in each radiator is chosen such that the total mass (and hence the total photoelectric absorption) is approximately the same in each case. Our data points are corrected for the efficiency of the crystal and the xenon counter, and for the transmission of the absorbing material (mainly helium) between the radiator and the spectrometer. Typical errors are indicated. They are essentially due to small changes in the electron beam and, mainly at low intensities, uncertainties in the background subtraction. For comparison with the experimental results, the curves in Fig. 3 show the shape of the expected distributions, calculated as discussed above. Our data points reproduce the calculated spectral shapes excellently. The maxima and minima due to interference effects are well resolved and appear exactly at the calculated frequencies. However, our overall intensity is somewhat lower than expected. In order to fit the experimental data better, the calculated spectra in Fig. 3 are scaled down by

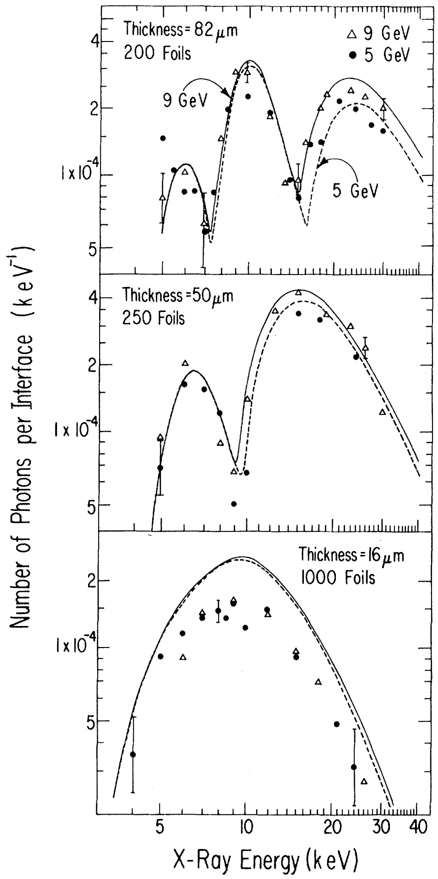

FIG. 3. Frequency spectra of transition radiation emerging from three radiators with different foil thicknesses. The points (solid points for 5-GeV and open symbols for 9-GeV electrons) are measured data; and the curves (dashed lines for 5-GeV and solid line for 9-GeV electrons) are the results of calculations (see text).

a constant factor of 0.64. We do not feel that this overall normalization indicates a strong departure from the expected behavior of transition radiation,[18] but we note that in other experiments the *total* yield of transition radiation has also been found to be lower than calculated.[4,6-8]

From Fig. 3, we see that the radiation hardens with increasing thickness of the radiator foils. This feature (which is also apparent in the data of Ref. 17) is of prime importance for practical applications, since it makes possible "tuning" of a transition-radiation system such that the radiation appears predominantly at frequencies where the detector is most sensitive. For high particle energies, one expects[4] the bulk of the radiation at frequencies close to the last maximum in the spectrum, near $\omega_{\max} = l_1\omega_1^2/2\pi c$ (where ω_1 is the plasma frequency of the foil material). This expression assumes the radiator to be transparent at $\omega_{\max}$, and it also assumes that $l_2/l_1 \gg 1$, i.e., that the spectrum is determined primarily by single-foil interference. For the 82- and 50-μm radiators, we measure the last maximum at 23 and 15 keV, respectively (see Fig. 3), while $\hbar\omega_{\max} = 29$ and 18 keV. This small but significant shift in frequency is caused by multifoil interference effects. In the case of the 16-μm radiator, with a measured maximum at 9 keV, reabsorption at low frequencies strongly influences the spectral shape, and leads to a suppression of the oscillatory structure at low frequencies.

Because of the interference phenomena, the transition-radiation yield saturates for high particle energies. In the present experiment, this saturation is expected at electron energies between 5 and 10 GeV.[4] Consequently very little energy dependence of our measured spectra is noticeable. The saturation energy should, however, increase slightly with increasing foil thickness, a tendency noticeable in our results around $\omega_{\max}$ for the thicker radiators.

Summarizing, the theoretical predictions of transition radiation, which are based on classical electromagnetic theory, are confirmed by our results in great detail. The radiation appears exactly at the expected frequencies; the structure of the spectrum is determined by the dimensions of the radiator; and the expected saturation at high particle energies is observed.

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[1]L. C. L. Yuan, C. L. Wang, H. Uto, and S. Prünster, Phys. Lett. 31B, 603 (1970).

[2]R. Ellsworth, J. MacFall, G. Yodh, F. Harris, T. Katsura, S. Parker, V. Peterson, L. Shiraishi, V. Stenger, J. Mulvey, B. Brooks, and J. Cobb, in *Proceedings of the Thirteenth International Conference*

on Cosmic Rays, Denver, Colorado, 1973 (Colorado Associated Univ. Press, Boulder, 1973), Vol. 4, p. 2819.

[3]J. Fischer, S. Iwata, V. Radeka, C. L. Wang, and W. J. Willis, Phys. Lett. 49B, 393 (1974).

[4]M. L. Cherry, G. Hartmann, D. Müller, and T. A. Prince, Phys. Rev. D 10, 3594 (1974).

[5]T. A. Prince, D. Müller, G. Hartmann, and M. L. Cherry, Nucl. Instrum. Methods 123, 231 (1975).

[6]L. C. L. Yuan, P. W. Alley, A. Bamberger, G. F. Dell, H. Uto, Nucl. Instrum. Methods 127, 17 (1975).

[7]C. Camps. V. Commichau, M. Deutschmann, H. Göddeke, K. Hangarter, W. Liesmann, U. Pützhofen, and R. Schulte, Nucl. Instrum. Methods 131, 411 (1975).

[8]A. I. Alikhanian, K. M. Avakina, G. M. Garibian, M. P. Lorikian, and K. K. Shikhliarov, Phys. Rev. Lett. 25, 635 (1970).

[9]M. L. Ter-Mikaelian, Nucl. Phys. 24, 43 (1961).

[10]G. M. Garibian, Zh. Eksp. Teor. Fiz. 60, 39 (1971) [Sov. Phys. JETP 33, 23 (1971)].

[11]M. L. Ter-Mikaelian, *High-Energy Electromagnetic Processes in Condensed Media* (Wiley, New York, 1972).

[12]For a single interface $d^2S_0/d\theta d\omega = 2\alpha\hbar\theta^3/\pi[(\gamma^{-2}+\theta^2+\omega_1^2/\omega_2)^{-1} - (\gamma^{-2}+\theta^2+\omega_2^2/\omega^2)^{-1}]^2$, where $\gamma = E/mc^2$, $\alpha = e^2/\hbar c$, and the dielectric constants are $\epsilon_{1,2} = 1-\omega_{1,2}^2/\omega^2$ with $\omega_{1,2}$ the plasma frequencies of the two media. {See G. M. Garibian, Zh. Eksp. Teor. Fiz. 39, 332 (1960) [Sov. Phys. JETP 12, 237 (1961)].}

[13]X. Artru, G. Yodh, and G. Mennessier, Phys. Rev. D 12, 1289 (1975).

[14]Explicitly, $Z_{1,2} = 4c/\omega(\gamma^{-2}+\theta^2+\omega_{1,2}^2/\omega^2)^{-1}$ (see Ref. 4).

[15]K. Hoshino, Y. Ohashi, A. Okada, K. Taira, and K. Yokoi, Acta. Phys. Acad. Sci. Hung. 29, Suppl. 4, 443 (1970).

[16]A. A. Frangian, F. R. Harutjunian, V. P. Kishinevski, A. A. Nazarian, and G. B. Torgomian, in Proceedings of the International Conference on Instrumentation for High Energy Physics, Dubna, U. S. S. R., 8–12 September 1970 (unpublished); see also Ref. 11, Sect. 29f.

[17]C. W. Fabjan and W. Struczinski, Phys. Lett. 57B, 483 (1975).

[18]The collimator (Fig. 1) absorbs x rays if emitted under angles much larger than the most probable angle ($\theta \approx 1/\gamma$). Although the radiation intensity drops very sharply with increasing θ, this effect accounts for part of the discrepancy between the measured and calculated intensity. (We appreciate a comment on this point by C. W. Fabjan and G. M. Garibian.)

Short-Range, High-Momentum Effects in the Reaction $^{16}O(\gamma, p_0)$ for E_γ=100–300 MeV

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The $^{16}O(\gamma,p_0)$ cross section has been measured for a series of photon energies between 100 and 300 MeV at proton angles of 45°, 90°, and 135°. Above 250 MeV, the results exceed simple shell-model predictions by several orders of magnitude. The data are compared with a calculation which involves Δ excitation in an intermediate state.

The (γ,p) reaction at energies well above the giant dipole resonance has long been recognized as a potential source of information about short-range effects in nuclei, on account of its sensitivity to high-momentum components in nucleon wave functions. If the experimental (γ,p) cross section is found to exceed that predicted by a shell-model calculation assuming a single-step knock-out mechanism, this could be taken as evidence that short-range effects are operating to increase the high-momentum amplitudes above those of the simple shell-model wave functions. Precise (γ,p) measurements[1,2] on several nuclei are now available for photon energies up to 100 MeV. The comparison of recent theoretical calculations with these data does not in fact yield any conclusive evidence that short-range effects are important. A consistent explanation of a wide range of data has been achieved[3] by a model which introduces a residual interaction with the range of the one-pion exchange force, thus increasing the cross section predicted by the single-step proton knock-out mechanism. However, the need for this medium-range residual interaction can be largely removed by an alternative and perhaps more reasonable choice[2] of the potential-